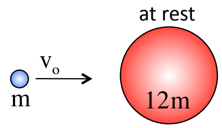
Problem 9.21

A neutron (mass "m") moving with velocity magnitude "v" strikes a stationary carbon atom (approximate mass "12m") in an elastic, head-on collision.

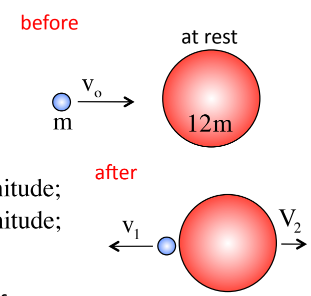
a.) What fraction of the neutron's kinetic energy is transferred to the carbon?



If you are feeling like an intellectual wimp (or if you don't have the time), you probably went to the textbook to find the formula that would do the job for you. If you were feeling more energetic, you derived it. I'll derive it.

We were told the collision was *elastic*. That means two things. First, it means that the *mechanical energy* is *conserved* through the collision. Second, and as a by-product of the first, what this *really* means is that the KINETIC ENERGY part of the mechanical energy equation is conserved (the collision will happen so quickly, no potential energy quantities will have time to change—if this isn't clear, think about how gravitational potential energy would change from *just before* to *just after* the collision). We also know that *momentum* is *conserved*, at least through the collision, because there are no indecently huge external impulses acting during that tiny collision interval.

To make life easier in a notational sense, I'm going to define the velocity magnitudes, as shown in the sketch. That is:



"v_o" is the neutron's initial velocity magnitude;
"v_1" is the neutron's after-collision velocity magnitude;
"V_2" as the carbon's after-collision velocity magnitude;

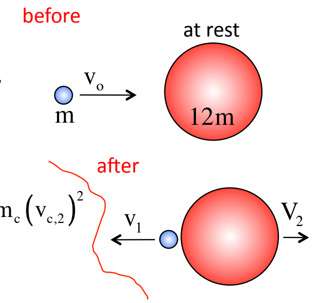
For momentum, noting that the neutron will undoubtedly *bounce back in the negative direction* after the collision, we can write:

$$\sum p_{1,x} + \sum \cancelto{0}{F_{external,x}\Delta t_{throughCollision}} = \sum p_{2,x}$$

$$\Rightarrow \quad m_n\vec{v}_{n,1} + \cancelto{0}{m_c\vec{v}_{c,1}} + 0 = m_n\vec{v}_{n,2} + m_c\vec{v}_{c,2}$$

$$\Rightarrow \quad m_n v_o = -m_n v_1 + m_c V_2$$

For the kinetic energy (again remembering that it is *kinetic energy* that is conserved through a collision), we can write:



$$\sum KE_1 = \sum KE_2$$

$$\Rightarrow \quad \frac{1}{2}m_n(v_{n,1})^2 + \frac{1}{2}\cancel{m_c}(\cancelto{0}{v_{c,1}})^2 = \frac{1}{2}m_n(v_{n,2})^2 + \frac{1}{2}m_c(v_{c,2})^2$$

$$\Rightarrow \quad \frac{1}{2}m_n v_o^2 = \frac{1}{2}m_n v_1^2 + \frac{1}{2}m_c V_2^2$$

In short, we have the following two equations to solve simultaneously:

$$m_n v_o = -m_n v_1 + m_c V_2$$

and

$$\frac{1}{2}m_n v_o^2 = \frac{1}{2}m_n v_1^2 + \frac{1}{2}m_c V_2^2$$

It turns out that these equations are not easily solved using the standard *substitution* method. Instead, we need to use trickery.

Specifically, if we group the equation parts by their *masses*, we get:

$$m_n v_o = -m_n v_1 + m_c V_2$$

$$\Rightarrow \quad m_n(v_o + v_1) = m_c V_2$$

and

$$\left(\cancel{\frac{1}{2}}\right)m_n v_o^2 = \left(\cancel{\frac{1}{2}}\right)m_n v_1^2 + \left(\cancel{\frac{1}{2}}\right)m_c V_2^2$$

$$\Rightarrow \quad m_n(v_o^2 - v_1^2) = m_c V_2^2$$

$$\Rightarrow \quad m_n(v_o - v_1)(v_o + v_1) = m_c V_2^2$$

Dividing the top relationship into the bottom relationship, we get:

$$\frac{\cancel{m_n}(v_o - v_1)\cancel{(v_o + v_1)}}{\cancel{m_n}\cancel{(v_o + v_1)}} = \frac{\cancel{m_c}V_2^2}{\cancel{m_c}V_2}$$

$$\Rightarrow \quad V_2 = (v_o - v_1)$$

This is a third, independent relationship between the variables.

Re-presenting the momentum relationship, then substituting into it, we get:

$$m_n v_o = -m_n v_1 + m_c V_2$$

$$\Rightarrow \quad V_2 = \frac{m_n(v_o + v_1)}{m_c}$$

With $V_2 = (v_o - v_1)$

and $V_2 = \dfrac{m_n(v_o + v_1)}{m_c}$

we can write:

$$(v_o - v_1) = \frac{m_n}{m_c}(v_o + v_1)$$

$$\Rightarrow \quad m_c v_o - m_c v_1 = m_n v_o + m_n v_1$$

$$\Rightarrow \quad v_1 = \frac{(-m_n + m_c)}{(m_c + m_n)} v_o$$

$$\Rightarrow \quad V_2 = (v_o - v_1) = v_o - \frac{(-m_n + m_c)}{(m_c + m_n)} v_o$$

IF YOU SKIPPED THE DERIVATION AND SIMPLY USED THE BOOK'S RELATIONSHIP, YOU SHOULD BE STARTING HERE!

Using this on our problem, we can write:

$$v_1 = \frac{(-m_n + m_c)}{(m_c + m_n)} v_o$$

$$= \frac{(-m + 12m)}{(12m + m)} v_o$$

$$= \frac{11}{13} v_o$$

and

$$V_2 = (v_o - v_1)$$

$$= v_o - \frac{11}{13} v_o$$

$$= \frac{2}{13} v_o$$

So now we can answer the question, which was, "What fraction of the neutron's KE is transferred to the carbon atom."

The original KE in the system is $\frac{1}{2} m v_o^2$. The carbons' final energy is:

$$KE_c = \frac{1}{2} m_c v_1^2$$

$$= \frac{1}{2}(12m)\left(\frac{2}{13} v_o\right)^2$$

$$= .142 m v_o^2$$

The ratio of the two is:

$$\frac{(KE_c)}{(KE_{initial,n})} = \frac{(.142 m v_o^2)}{\left(\frac{1}{2} m v_o^2\right)}$$

$$= .284$$

Apparently, 28.4% of the energy is absorbed by the carbon with the remaining 71.6% staying with the neutron. (NOTE: This is how nuclear accelerators slow neutrons down to give them a better chance of interacting with the fissionable materials to get a nuclear reaction.)

b.) If the neutron's initial KE was 1.60×10^{-13} J, what would the carbon and neutron "final" KE be after the collision?

For the carbon:

$$KE_c = .284(KE_o)$$

$$= .284(1.60 \times 10^{-13}\ J)$$

$$= 4.54 \times 10^{-14}\ J$$

For the neutron:

$$KE_c = .716(KE_o)$$

$$= .716(1.60 \times 10^{-13}\ J)$$

$$= 1.15 \times 10^{-13}\ J$$